NATIONAL SENIOR CERTIFICATE EXAMINATION
NOVEMBER 2023

MATHEMATICS: PAPER I

MARKING GUIDELINES

Time: 3 hours | 150 marks

These marking guidelines are prepared for use by examiners and sub-examiners, all of whom are required to attend a standardisation meeting to ensure that the guidelines are consistently interpreted and applied in the marking of candidates' scripts.

The IEB will not enter into any discussions or correspondence about any marking guidelines. It is acknowledged that there may be different views about some matters of emphasis or detail in the guidelines. It is also recognised that, without the benefit of attendance at a standardisation meeting, there may be different interpretations of the application of the marking guidelines.

SECTION A

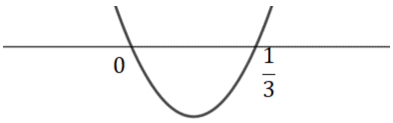
QUESTION 1

(a) (1) $x = \log_3 2$

(2) $$x + 7 = (x+1)^2$$
$$x^2 + x - 6 = 0$$
$$(x+3)(x-2) = 0$$
$$x \neq -3 \text{ or } x = 2$$

(b) $$y = 2x + 7$$
$$x^2 - x - (2x+7) - 3 = 0$$
$$x^2 - 3x - 10 = 0$$
$$(x-5)(x+2) = 0$$
$$x = 5 \text{ or } x = -2$$
$$(5\,;17) \text{ or } (-2\,;3)$$

(c) $$3x^2 - x \leq 0$$
$$x(3x-1) \leq 0$$



$$0 \leq x \leq \frac{1}{3}$$

QUESTION 2

(a) $(0;32)$

(b) $f(x)=(x+2)(x^2-8x+16)$

$f(x)=x^3-6x^2+32$

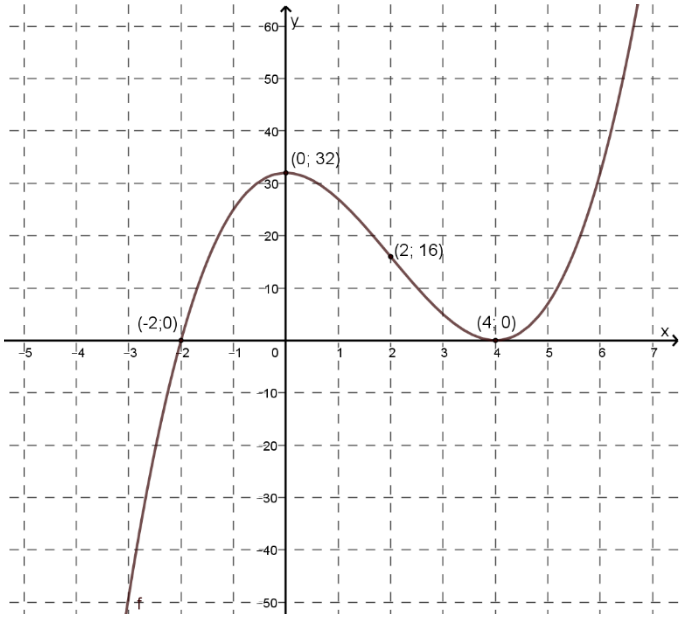
$f'(x)=3x^2-12x$

(c) $3x^2-12x=0$ (no mark as this is stated in question)

$3x(x-4)=0$

$x=0$ or $x=4$

(d) Turning points
X intercept
Point of inflection
Shape



(e) $x \geq -2$

QUESTION 3

(a) $1 = \log_m(3+2)$

$m = 5$

(b) $y = \log_5(0+2)$

$C(0\,;0{,}4)$

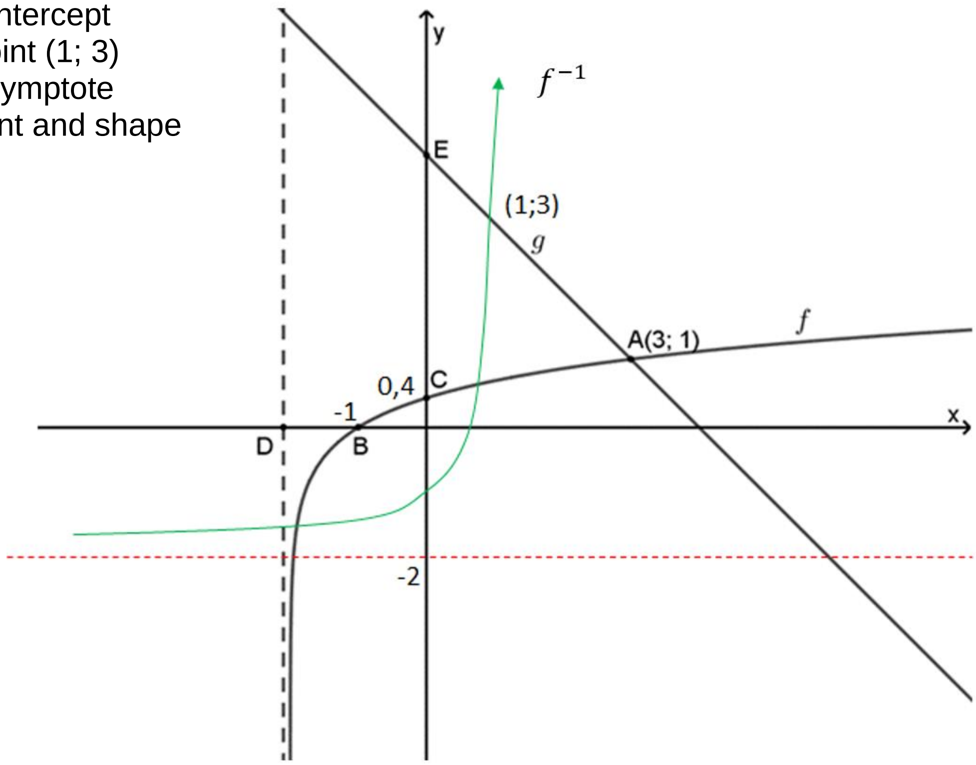
$0 = \log_5(x+2)$

$x = -1$

$B(-1;0)$

(c) $x \in (-2\,;3]$ notation Alt: $-2 < x \leq 3$

(d) y intercept
Point (1; 3)
Asymptote
x-int and shape



QUESTION 4

(a) (1)

$$f(x) = \sqrt[5]{x^2} - \frac{3}{x^3}$$

$$f(x) = x^{\frac{2}{5}} - 3x^{-3}$$

$$f'(x) = \frac{2}{5}x^{\frac{-3}{5}} + 9x^{-4}$$

(2)

$$g(x) = \frac{(x-1)(x-1)}{4(x-1)}$$

$$g(x) = \frac{x}{4} - \frac{1}{4}$$

$$g'(x) = \frac{1}{4}$$

(b)

$y = 7x + c$ Alt: $y - 5 = 7(x - 4)$

$5 = 7(4) + c$

$c = -23$

$y = 7x - 23$

(c)

$$h'(x) = \lim_{h \to 0} \frac{5(x+h) - 8 - (5x - 8)}{h}$$

$$h'(x) = \lim_{h \to 0} \frac{5x + 5h - 8 - 5x + 8}{h}$$

$$h'(x) = \lim_{h \to 0} \frac{5h}{h}$$

$$h'(x) = 5$$

QUESTION 5

(a)
$$T_n = 22 + (n-1)(3)$$
$$T_n = 3n + 19$$
$$3n + 19 = 262$$
$$3n = 243$$
$$n = 81$$

(b)
$$\sum_{n=1}^{81} 3n + 19$$

(c)
$$\frac{n}{2}\left(2(22) + (n-1)(3)\right) > 15\,000$$
$$3n^2 + 41n - 30\,000 > 0$$
$$n < -107{,}1 \text{ or } n = 93{,}4$$
$$n = 94 - 81 = 13$$

QUESTION 6

(a)

Investment 1: $$\frac{3\,500\left(\left(1+\frac{0,15}{12}\right)^{60}-1\right)}{\frac{0,15}{12}}=\text{R}310\,010,78$$

Investment 2: $$24\,000\left(1+\frac{0,20}{4}\right)^{20}+7\,000\left(1+\frac{0,20}{4}\right)^{8}=\text{R}74\,021,33$$

The total of the lump sum is R384 032,11

(b) (1)

$$250\,000=\frac{x\left(1-\left(1+\frac{0,10}{12}\right)^{-120}\right)}{\frac{0,10}{12}}$$

$$x=\text{R}3\,303,77$$

(2)

$$250\,000\left(1+\frac{0,10}{12}\right)=\frac{6\,000\left(1-\left(1+\frac{0,10}{12}\right)^{-n}\right)}{\frac{0,10}{12}}$$

$$\frac{1\,123}{1\,728}=\left(1+\frac{0,10}{12}\right)^{-n}$$

$$-n=\log_{\left(1+\frac{0,10}{12}\right)}\frac{1\,123}{1\,728}$$

$$-n=-51,93$$

$$n=52\text{ months}$$

SECTION B

QUESTION 7

(a) $x^2 - 16x + 64 + 9$

$(x-8)^2 + 9$

$m = 8$ and $p = 9$

(b) (1) k^2

(2) $k^{\frac{1}{3}}$ or $\sqrt[3]{k}$

(3) $\dfrac{2}{k}$ or $2k^{-1}$

(c)
$$\frac{3^{\log_p 2}}{3^{\log_p 8}} = 9$$

$$3^{\log_p 2} = 3^{\log_p 8}.3^2$$

$$\log_p 2 = \log_p 8 + 2$$

$$\log_p 2 - \log_p 8 = 2$$

$$\log_p \frac{1}{4} = 2$$

$$p = \frac{1}{2}$$

Alternative:

$3^{\log_p 2 - \log_p 2^3} = 3^2$

$\therefore \log_p 2 - 3\log_p 2 = 2$

$-2\log_p 2 = 2 \quad \therefore \log_p 2 = -1 \quad \therefore p^{-1} = 2 \qquad \therefore p = \dfrac{1}{2}$

QUESTION 8

(a) $y = x$

(b) $x = \frac{12}{x}$

$x^2 = 12$

$x = \pm\sqrt{12}$

$B\left(\sqrt{12}\,;\sqrt{12}\right)$

(c) $f'(x) = -12x^{-2}$ or $f'(x) = \frac{-12}{x^2}$

$\frac{-12}{x^2} = \frac{-1}{3}$

$-36 = -x^2$

$x^2 = 36$

$x = \pm 6$

$x = 6$

$C(6\,;2)$

$y = -\frac{1}{3}x + c$ or Alt. for equation line: $y - 2 = -\frac{1}{3}(x - 6)$ $\therefore y = -\frac{1}{3}x + 4$

$2 = -\frac{1}{3}(6) + c$

$c = 4$

Area of $\Delta ACO = \frac{1}{2} \times 4 \times 6$

Area of $\Delta ACO = 12$ units^2

QUESTION 9

(a) $y = a(x-1)^2 + 2$

$D(7\,;5)$

$5 = a(7-1)^2 + 2$

$3 = 36a$

$a = \frac{1}{12}$

(b) $y = \frac{1}{100}(x-p)^2 + 4$

$B(-5\,;5)$

$1 = \frac{1}{100}(-5-p)^2$

$100 = 25 + 10p + p^2$ or $(p+5)^2 = 100$ $\therefore p+5 = \pm 10$

$p^2 + 10p - 75 = 0$

$(p+15)(p-5) = 0$

$p \neq -15$ or $p = 5$

The distance between the poles should be 20 metres.

QUESTION 10

(a) (1) $T_n = 1.(1{,}1)^{24-1}$

$T_n = 8{,}95$ or $9{,}0$ litres/minute ***9,0 is the rounded off value***

(2) $S_n = \dfrac{60.(1{,}1^{24} - 1)}{1{,}1 - 1}$

$S_n = 5\ 309{,}8$ litres

(b) $T_2 - T_1 = 4\,; T_3 - T_2 = 7$ and $T_{10} = 0$

$3a + b = 4$

$2a = 3$

$a = \dfrac{3}{2}$ and $b = -\dfrac{1}{2}$

$0 = \dfrac{3}{2}(10)^2 - \dfrac{1}{2}(10) + c$

$c = -145$

$T_n = \dfrac{3}{2}n^2 - \dfrac{1}{2}n - 145$

(c) $\dfrac{r}{1-r} = \dfrac{1}{5}$ Alt. solution: $\dfrac{a}{1-a} = \dfrac{1}{5}$ $\therefore 5a = 1 - a$ $\therefore a = \dfrac{1}{6}$

$5r = 1 - r$

$r = \dfrac{1}{6}$

but $a = r$, hence $a = \dfrac{1}{6}$

(d) $x\,;y\,;x+y\,;...$

$\frac{x+y}{y}=\frac{y}{x}$ ✓ ALT. $a=x$ and $r=\frac{y}{x}$ hence $T_3=ar^2=x\left(\frac{y}{x}\right)^2=\frac{y^2}{x}$

But $T_3=\frac{y^2}{x}=x+y$ and $y^2=x^2+xy$

$$x^2+xy=y^2$$

$$x^2+xy-y^2=0$$

$$x=\frac{-(y)\pm\sqrt{(y)^2-4(1)(-y^2)}}{2(1)}$$

$$x=\frac{-(y)\pm\sqrt{5}y}{2(1)}$$

$$x=\frac{y\left(-1\pm\sqrt{5}\right)}{2}$$

$\frac{x}{y}=\frac{-1+\sqrt{5}}{2}$ as x and y are both positive

QUESTION 11

(a) (1) (i) 9^6 (First password) or 531 441

(ii) $9^6 \times 8 \times 7 \times 6 \times 4 = 714\ 256\ 704$
(The four digit even password *x* first a password)

(2) $4 \times 7! \times 1$ (Ends in a 3 and starts with a 6, 7, 8 or 9)
$3 \times 7! \times 2$ (Ends with a 6 or 9, starts with 7, 8 and either 6 or 8)
$= 4 \times 7! \times 1 + 3 \times 7! \times 2 = 50\ 400$
ALT: $10 \times 7!$

(b) (1) $\frac{3}{36}$ or $\frac{1}{12}$

All *y* intercepts are negative.
Only three values less than –29.

	1	2	3	4	5	6
1	-1	-2	-3	-4	-5	-6
2	-2	-4	-6	-8	-10	-12
3	-3	-6	-9	-12	-15	-18
4	-4	-8	-12	-16	-20	-24
5	-5	-10	-15	-20	-25	-30
6	-6	-12	-18	-24	-30	-36

(2) 1 (They will always be real, rational and unequal)

(3) The shape of the graph is

For non-real roots the turning point needs to be below the *x*-axis.

$= \frac{2}{6}$ or $\frac{1}{3}$

Alternate Solution:
$h(x) = -x^2 + 2gx + g^2 + 4 - r$
$\Delta = (-2g)^2 - 4(-1)(-g^2 + 4 - r)$
$= 4g^2 - 4g^2 + 16 - 4r$
$\therefore \Delta = 16 - 4r < 0 \qquad \therefore > 4$
$\therefore 5$ or 6 hence $P(\text{non-real roots}) = \frac{2}{6} = \frac{1}{3}$

QUESTION 12

(a) $y = -\frac{2}{5}x + 6$

(b)

$$\text{Volume} = x(x-2)\left(\frac{-2}{5}x + 6\right)$$

$$\text{Volume} = x\left(\frac{-2}{5}x^2 + 6x + \frac{4}{5}x - 12\right)$$

$$\text{Volume} = \frac{-2}{5}x^3 + 6x^2 + \frac{4}{5}x^2 - 12x$$

$$\frac{Dv}{dx} = -\frac{6}{5}x^2 + 12x + \frac{8}{5}x - 12$$

$$-\frac{6}{5}x^2 + 12x + \frac{8}{5}x - 12 = 0$$

$$-6x^2 + 60x + 8x - 60 = 0$$

$$-6x^2 + 68x - 60 = 0$$

$$3x^2 - 34x + 30 = 0$$

$$x = 0{,}96 \text{ or } x = 10{,}3689$$

Vertical beam

$$\text{Height of vertical beam} = -\frac{2}{5}(10{,}3689...) + 6$$

Height of vertical beam for maximum volume is 1,9 metres.

Total: 150 marks